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International Journal of Heat and Mass Transfer 47 (2004) 653–658
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Development of a new Biot number and lag factor correlation for drying applications

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Received 14 April 2003; received in revised form 11 August 2003

Abstract

The present paper deals with the development of a new Biot number–lag factor (*Bi*–*G*) correlation for drying applications. Development of this correlation is based on the experimental data acquired from various sources in the literature. Using the developed correlation, moisture transfer parameters such as moisture diffusivity and moisture transfer coefficient for three regular shaped objects, e.g. slab, cylinder and sphere are calculated and compared with the experimental moisture content variations. The results showed an appreciably high agreement between the measured and predicted moisture content values from the correlation. Hence, the present correlation is considered as a useful tool for practical drying applications and a good contribution to the state-of-art of drying.

1. Introduction

Drying is a widely used industrial process, consuming 7–15% of total industrial energy production in the industrialized world [1]. The major factors affecting the moisture transport during solids drying can be classified as

- External factors: these are the factors related to the properties of the surrounding air such as temperature, pressure, humidity, velocity and area of the exposed surface.
- Internal factors: these are the parameters related to the properties of the material (or product) such as moisture diffusivity, moisture transfer coefficient, water activity, structure and composition, etc.

Moisture diffusivity is an important transport property needed for accurate modeling in food drying applications and is generally assumed as an independent of moisture transfer path [2,3]. Its accurate determination can lead to better design of drying systems and optimization of drying processes for the respective applications. Although numerous theoretical and experimental works have been directed towards moisture diffusivity estimation [3–5], limited data are available on moisture diffusivity, with a wide variation due to the structure complexity of foods and different methods of its estimation [6]. Although the authors have recently developed some drying correlations (e.g., *Bi*–*S*, *Bi*–*Re* and *Bi*–*Di*) [7–10] for practical drying applications, in the present work one of the most significant drying process parameters, lag factor, and its connection with the Biot number have been studied as a separate, new work. In this regard, the objective of the present study is to develop a new model (so-called Biot number–lag factor correlation) to estimate the moisture transfer parameters and hence moisture content variations and to validate the applicability of the present correlation with the measured moisture distributions for three types of products obtained from the literature [11–13].

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doi:10.1016/j.ijheatmasstransfer.2003.08.006

Nomenclature

a, b	constants
A, B	constants
Bi	Biot number for moisture transfer
D	moisture diffusivity (m^2/s)
Fo	Fourier number for moisture transfer
G	lag factor
k	moisture transfer coefficient (m/s)
M	moisture content (kg of water/kg of dry material)
S	drying coefficient (1/s)
t	time (t)
Y	characteristic dimension (half thickness for slab, radius for cylinder and sphere) (m)
Greek symbols	
Φ	dimensionless moisture content
μ	root of the characteristic equation
Subscripts	
e	equilibrium
i	initial
1	first characteristic value

2. Analysis

During solids drying, transient moisture diffusion process occurs, which is similar in form to the Fourier law of heat conduction. The dimensionless moisture content values with time can be expressed in exponential form as follows, as defined earlier [1]:

$$\Phi = G\exp(-St) \tag{1}$$

where G is the lag factor, indicating the internal resistance in the wet solid to moisture transfer during drying and S is drying coefficient (1/s), representing the drying capability of the wet solid.

The dimensionless moisture (Φ) content can be represented in terms of moisture content at any point of the solid object as

$$\Phi = (M - M_e)/(M_i - M_e) \tag{2}$$

The Biot number for moisture transfer can be defined as a function of moisture transfer coefficient, moisture diffusivity and characteristic dimension as

$$Bi = kY/D \tag{3}$$

The moisture diffusivity (D) relation developed earlier [1] for slab, cylindrical and spherical objects is given as

$$D = (SY^2)/(\mu_1^2) \tag{4}$$

The moisture transfer coefficient for slab, cylindrical and spherical product is obtained from Biot number definition as

$$k = (D \times Bi)/Y \tag{5}$$

In the present study we develop a new Biot number–lag factor (Bi–G) correlation so as to calculate Biot number directly from the correlation which in turn results in calculating the moisture transfer parameters from the above equations in a simple and accurate manner. The correlation is developed in the following form:

$$Bi = aG^b \tag{6}$$

where a and b are constants.

3. Results and discussion

This section presents the methodology for evaluating the drying process and moisture transfer parameters and discuss the development of Biot number–lag factor correlation.

The procedure employed in estimating the process parameters is as follows:

- The experimental moisture content values are non-dimensionalized using Eq. (2).
- The dimensionless moisture content values versus drying time are regressed in the exponential form of Eq. (1) using the least square curve-fitting method. Hence lag factor (G) and drying coefficient (S) appear in the exponential equation.
- The characteristic roots (μ_1's) which appear in the moisture diffusivity relation (Eq. (4)) are determined using the newly developed expressions as follows [10]:
  For slab shapes,
  $$\mu_1 = -419.24G^4 + 2013.8G^3 - 3615.8G^2 + 2880.3G - 858.94 \tag{7}$$
  For cylindrical shapes,
  $$\mu_1 = -3.4775G^4 + 25.285G^3 - 68.43G^2 + 82.468G - 35.638 \tag{8}$$
  For spherical shapes,
  $$\mu_1 = -8.3256G^4 + 54.842G^3 - 134.01G^2 + 145.83G - 58.124 \tag{9}$$
- The moisture diffusivity values are then calculated using Eq. (4).

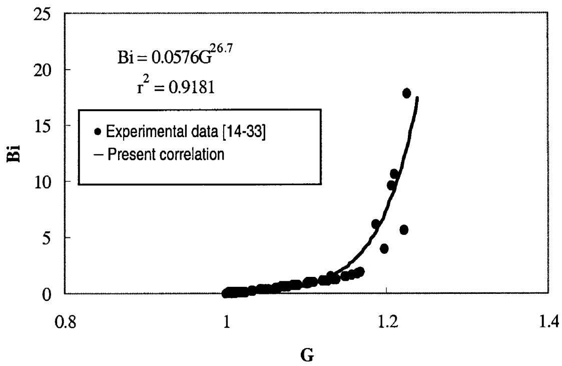

Fig. 1. *Bi–G* diagram for food products subjected to drying.

- Using the experimental drying data taken from many literature sources [14–33] for different geometrical shaped products (e.g. slab, cylinder, sphere cubic, etc.), we have obtained the Biot number–lag factor correlation for several kinds of food products subjected to drying (Fig. 1) with a correlation coefficient of 0.9181 as

$$Bi = 0.0576G^{26.7} \tag{10}$$

- Finally, the moisture transfer coefficients are calculated using Eq. (5).

4. Illustrative example

The prime objective of this example is to show how the present correlation can be utilized in determining the moisture transfer parameters using the existing experimental moisture data of a solid product. In order to explain the application of the correlation, we will use literature experimental moisture measurements of onion slices, high amylose starch powder and yam as slab, cylinder and sphere, respectively. The thermophysical properties of materials employed in the experiments are given in Table 1.

The following procedure is employed to determine the moisture transfer parameters and dimensionless moisture distribution:

- Lag factor is determined by regressing the experimental dimensionless moisture content values against the drying time in the form of Eq. (1) using the least square curve fitting method (with correlation coefficients of 0.982, 0.997 and 0.999), and found as 1.1503, 1.0181 and 1.2864 for slab, cylinder and sphere, respectively.
- Using the present correlation (Eq. (10)), the Biot numbers are estimated as 2.4214, 0.0929 and 47.9471 for slab, cylinder and sphere, respectively.
- The characteristic roots are evaluated using the relations in Eqs. (7)–(9) for three respective products.
- The moisture diffusivities are then calculated using the Eq. (4).
- The moisture transfer coefficients are calculated using Eq. (5).
- The dimensionless moisture distributions for three shapes of products are given in a simplified form as (for details, see [1]):

$$\Phi = A_1 B_1 \tag{11}$$

where

$$\text{for slab:} \quad A_1 = G = \exp((0.2533Bi)/(1.3 + Bi)) \tag{12}$$

$$\text{for cylinder:} \quad A_1 = G = \exp((0.5066Bi)/(1.7 + Bi)) \tag{13}$$

$$\text{for sphere:} \quad A_1 = G = \exp((0.7599Bi)/(2.1 + Bi)) \tag{14}$$

$$\text{and for all objects:} \quad B_1 = \exp(-\mu_1^2 Fo) \tag{15}$$

where

$$Fo = Dt/Y^2 \tag{16}$$

The values of the drying coefficient (S), lag factor (G), Biot number (Bi), root of the characteristic equation (μ), moisture diffusivity (D) and moisture transfer coefficient (k) for the slab, cylindrical and spherical products were obtained using the above listed methodology and these are tabulated in Table 2. It is important to emphasize that the moisture content data of these three products were not employed in the development of the present

Table 1
Thermophysical properties of experimental data

Shape	Slab	Cylinder	Sphere
Air temperature	50 °C (323.15 K)	332 K	105 °C (378.15 K)
Air relative humidity	–	–	11%
Air velocity	0.5 m/s	2 m/s	–
Characteristic dimension (Y)	0.0025 m (half thickness)	0.005 m (radius)	0.03 m (radius)
References	[13]	[11]	[12]

Table 2
Obtained drying process and moisture transfer parameters for the samples

Parameters	Products		
	Slab (onion slice)	Cylinder (high amylose starch powder)	Sphere (yam)
S (1/s)	0.0002	0.0006	0.0046
G	1.1503	1.0181	1.2864
Bi	2.4214	0.0929	47.9471
μ_1	0.9951	0.3398	1.6552
D (m^2/s)	1.2623×10^{-9}	1.2991×10^{-7}	1.511×10^{-6}
k (m/s)	1.2226×10^{-6}	2.4137×10^{-7}	2.4151×10^{-3}

correlation to avoid any bias. Thus, these can help determine the applicability and accuracy of the present correlation.

Here, we now verify applicability of the Bi–G correlation. In this regard, using the obtained data in Table 2 we calculate the dimensionless average moisture content profiles for these slab, cylindrical and spherical products subject to drying at different conditions and compare these calculated dimensionless moisture content profiles with the experimental dimensionless moisture content values. Both calculated and experimental profiles are shown in Figs. 2–4. The maximum and average errors between predicted and measured moisture content values were found to be ±27.8% and ±12.7% for slab, ±5.78% and ±2.43% for cylinder and ±18.89% and ±3.08% for sphere with respect to measured values, respectively. The agreement between the calculated values and experimental data, based on the maximum error, is good for slab, excellent for cylinder, very good for sphere. Note that at $t = 0$, the value of correlated dimensionless moisture content becomes more than 1, because of the nature of Eq. (1). In fact, this was expected due to the fact that it shows the value of lag factor (G), referring to the boundary condition of third kind in transient moisture transfer. Note that if the lag factor is equal to 1, we would simplify to lumped capacitance method.

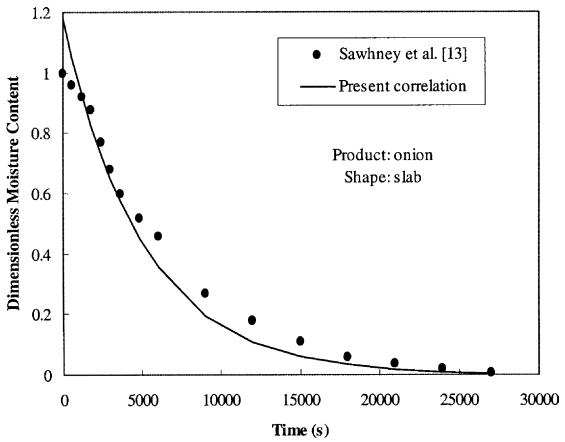

Fig. 2. Measured and calculated dimensionless center moisture distribution of slab.

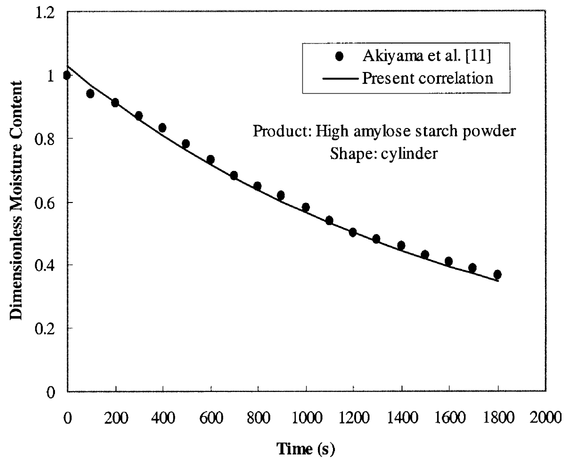

Fig. 3. Measured and calculated dimensionless center moisture distribution of cylinder.

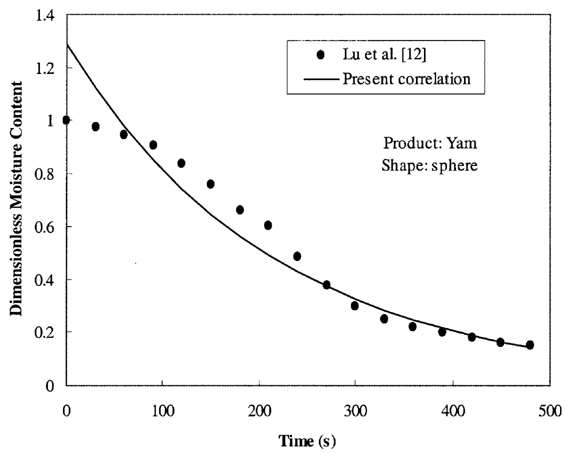

Fig. 4. Measured and calculated dimensionless center moisture distribution of sphere.

5. Conclusions

The development of a new drying correlation (*Bi–G*) correlation for products subjected to drying has been presented in this paper. The application of the present correlation is explained through an illustrative example to determine the moisture transfer parameters i.e., moisture diffusivity and moisture transfer coefficients. Moreover, dimensionless center moisture distributions were obtained for slab, cylinder and sphere. The obtained moisture distribution profiles were compared with the experimental data and found to be in good agreement with it. Thus the present correlation can be used with reasonable accuracy and confidence for such drying applications.

Acknowledgements

The authors acknowledge the support provided by King Fahd University of Petroleum & Minerals (KFUPM) for this work under the KFUPM research grant # ME/ENERGY/203/.

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